

① PRAVUGLI TROUGAO ČIJA JE JEDNA KATETA 15 cm
A HIPOTENUSA 25 ROTIRA OKO KRAĆE KATETE.

ODREDITI P I V TAKO NASTALOG TELA

② PRAVUGLI TRAPEZ OSNOVICA 5 cm i 2 cm
i VISINE 4 cm. ROTIRA OKO DUGE OSNOVICE.

ODREDITI P I V TAKO DOBIJENOG TELA.

③ JEDNAKOKRAKI TRAPEZ OSNOVICA 15 cm i 3 cm
i VISINE 8 cm ROTIRA OKO KRAĆE OSNOVICE.

ODREDITI P I V TAKO DOBIJENOG TELA.

④ JEDNAKOKRAKI TRAPEZ OSNOVICA 8 cm i 4 cm
SA OŠTRIM UGLOM 45° ROTIRA OKO DUGE OSNOVICE.

ODREDITI P I V TAKO DOBIJENOG TELA.

⑤ DIJAGONALE ROMBA SU 12 cm i 16 cm
Izračunati P i V TELA KOJE NASTAJE
ROTACIJOM ROMBA OKO DUGE DIJAGONALE.

⑥ ROMB ČIJE SU DIJAGONALE 6 cm i 8 cm
ROTIRA OKO JEDNE (KRAĆE) STRANICE. ODREDITI

P I V TAKO DOBIJENOG TELA.

⑦ PRAVUGLI TROUGAO ČIJE SU KATETE 3 cm i
4 cm ROTIRA OKO HIPOTENUZE. ODREDITI

P I V TAKO DOBIJENOG TELA.

⑧ PRAVILAN TROKUT SA STRANICE 2 cm ROTIRA
OKO DUGE DIJAGONALE. ODREDITI P I V TAKO
DOBIJENOG TELA.

⑨ JEDNAKOKRAKI TROUGAO ČIJI JE KRAK 10 cm
i UGLOM IZMEĐU KRAKOVA 30° ROTIRA OKO KRAKA.

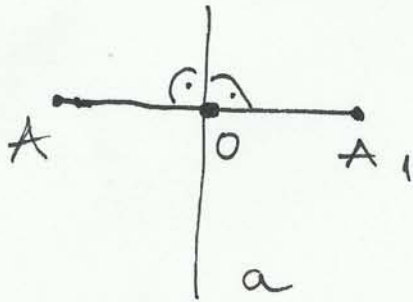
ODREDITI P i V TAKO DOBIJENOG TELA.

(10) PRAVOUGLI TRAPEZ ROTIRA OKO PRAVE
KOJU PRIPADA KRAĆI KRAK. IZRAČUNATI P i V
TAKO DOBIJENOG TELA AKO SU DUEINE
OSNOVICA 10 cm i 6 cm i OSTAR UGAO 60° .

(11) PRAVOUGLI STRANICE $4\sqrt{3}\text{ cm}$ SA OSTRIM UGLOM
 60° ROTIRA OKO DUE DIJAGONALE. ODREDITI
IZPREMINU TAKO DOBIJENOG TELA.

(12) IZRAČUNATI IZPREMINU TELA KOJE NASTAJE
ROTACIJOM PRAVOUGAONIKA STRANICA 2 cm i 5 cm
OKO PRAVE PARALELNE DUEOS STRANICI PRAVO-
UGAONIKA KOJA SE NALAZI NA RASTOJANJU 1 cm
OD NJE.

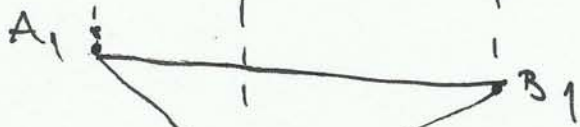
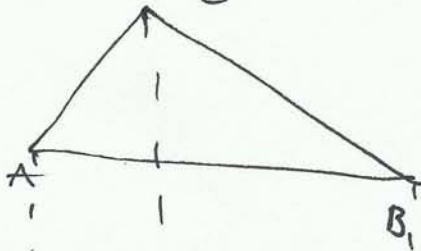
OSNA SIMETRIJA



$$AO = OA_1$$

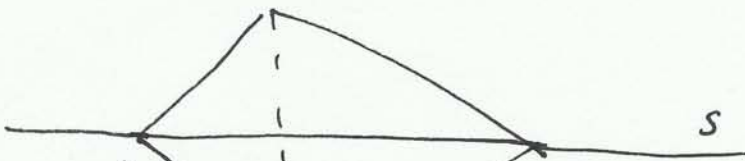
a

c



c1

c



A=A1

B=B1

c1

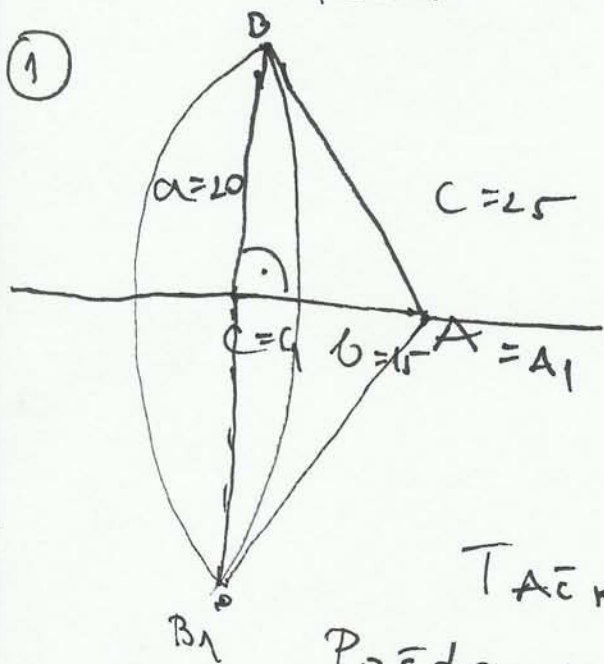
S

Preslikavanje
tačka po tačku.
Kada ih spojimo
dobijemo trougao.

A i B su fiksne
tačke jer
se nalaze na
osi S.

ROTACIJA

①



Rešavamo
figuru koju
imamo!

$$b = 15$$

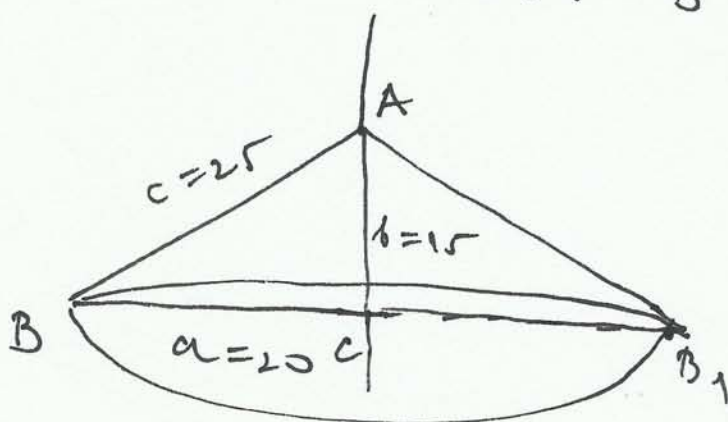
$$c = 25$$

$$a^2 = c^2 - b^2$$

$$a^2 = 25^2 - 15^2 = 625 - 225 = 400$$

$$a = 20$$

Tačka koja rotira pravi KUPU.
Pošto rotira oko AC ose
tačke A i C su fiksne,
samo se tačka B preslikava u B₁



Uvek se lakše nacrtati sliku tako
da se osa rotacije normalna na
kraću stranu papira.
Sada se jasno vidi da smo dobili
KUPU.

KUPA

$$r = a = 20$$

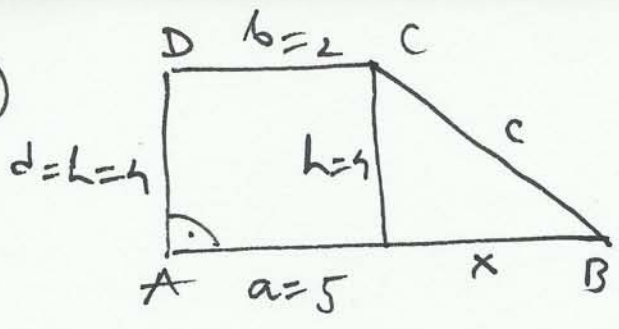
$$H = b = 15$$

$$S = c = 25$$

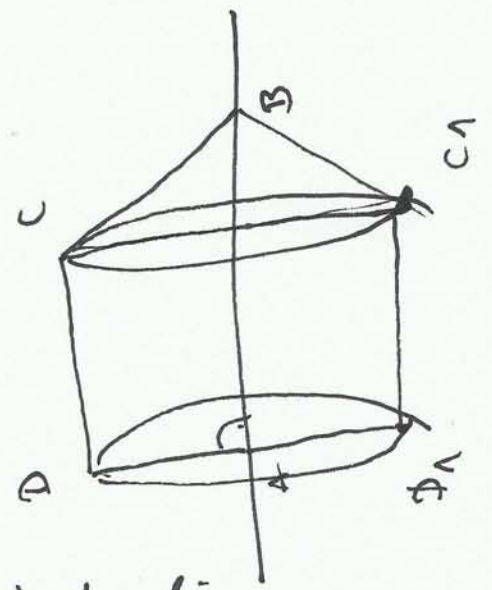
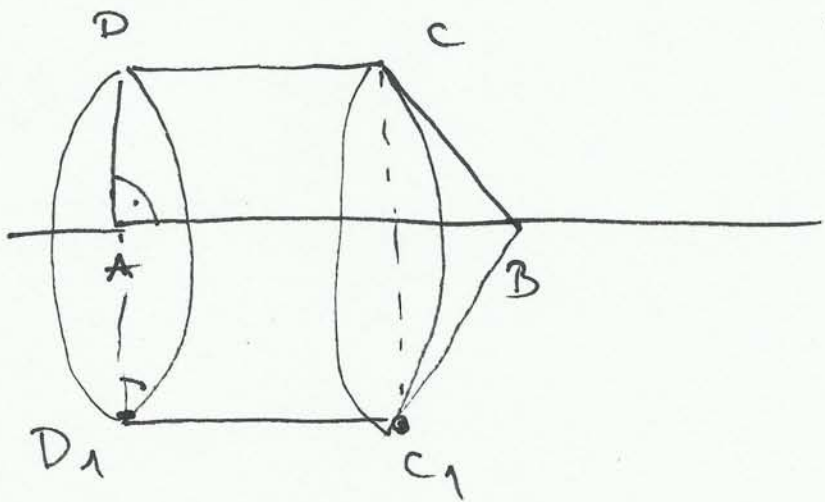
$$P = r^2 \pi + r \pi S = 20^2 \pi + 20 \pi 25 = 900 \pi \text{ cm}^2$$

$$V = \frac{r^2 \pi H}{3} = \frac{20^2 \pi 15}{3} = 1200 \pi \text{ cm}^3$$

2



Prvo rešimo figuru
 Na dno 2^oj s_{ve}
 elemente
 $x = a - b = 5 - 2 = 3$
 $c^2 = h^2 + x^2$
 $c^2 = 4^2 + 3^2$
 $c^2 = 25$
 $c = 5$



A i B su fiksne
 jer su na osi

Jasno se vidi da smo dobili

Kup u ;

V A₁ AK

$r_k = h = 4$

$r_v = h = 4$

$H_k = x = 3$

~~$H_v = b = 2$~~

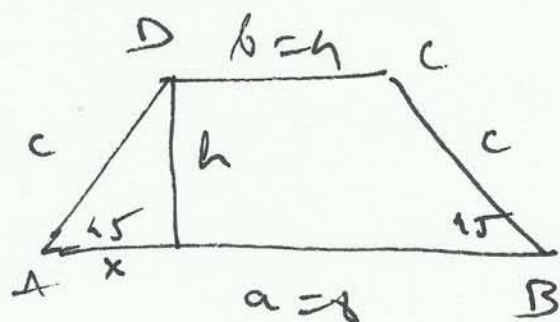
$s_k = c = 5$

$H_v = b = 2$

$P = B + M_v + M_k = r_v^2 \pi + 2r_v \pi H_v + r_k \pi s_k =$
 $= 4^2 \pi + 2 \cdot 4 \pi \cdot 2 + 4 \cdot \pi \cdot 5 = 16\pi + 16\pi + 20\pi$
 $P = 52\pi$

$V = V_v + V_k = \frac{1}{3} r_v^2 \pi H_v + \frac{r_k^2 \pi H_k}{3} = 2 \cdot 4^2 \pi +$
 $+ \frac{4^2 \pi \cdot 3}{3} = 32\pi + 16\pi = 48\pi$

4

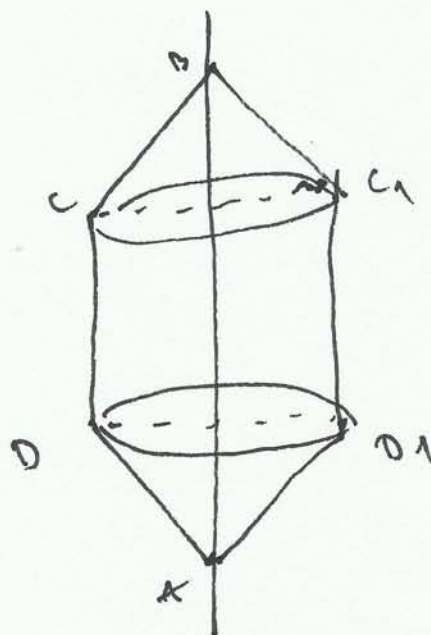
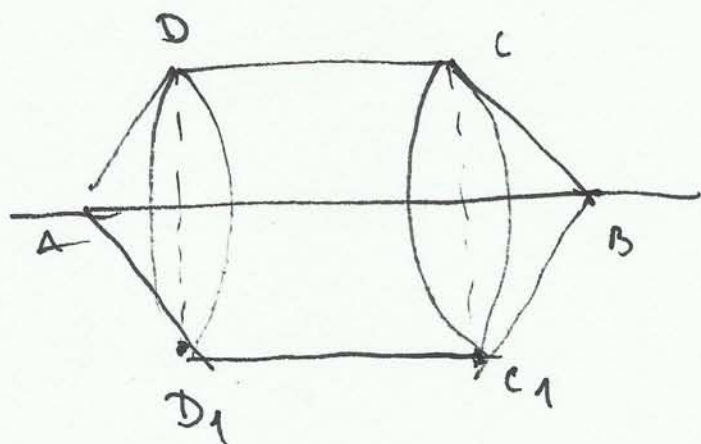


Resolvamos figura:

$$x = \frac{a-b}{2} = \frac{8-4}{2} = 2$$

$$h = x = 2$$

$$c = x\sqrt{2} = 2\sqrt{2}$$



Tačke koje rotiraju su C i D

A i B su fiksne tačke.

Doliki smo dve iste KUPE i VAČAK

$$r_k = h = 2$$

$$H_k = x = 2$$

$$s_k = c = 2\sqrt{2}$$

$$r_v = h = 2$$

$$H_v = a - b = 4$$

$$P = 2M_k + M_v = 2r_k \pi s_k + 2r_v \pi H_v =$$

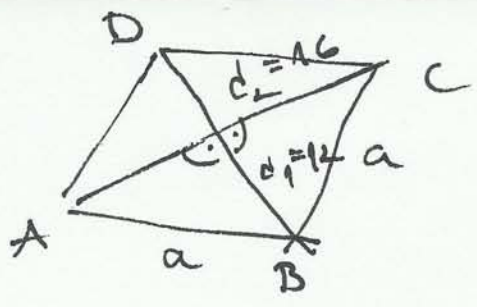
$$= 2 \cdot 2\pi \cdot 2\sqrt{2} + 2 \cdot 2 \cdot \pi \cdot 4 = (8\sqrt{2} + 16)\pi$$

$$P = 8\pi(\sqrt{2} + 2)$$

$$V = 2V_k + V_v = 2 \frac{r_k^2 \pi H_k}{3} + r_v^2 \pi H_v =$$

$$= 2 \frac{2^2 \pi \cdot 2}{3} + \cancel{2}^2 \pi \cdot 4 = \frac{16 + 48}{3} \pi = \frac{64}{3} \pi$$

5



Resimo figuru

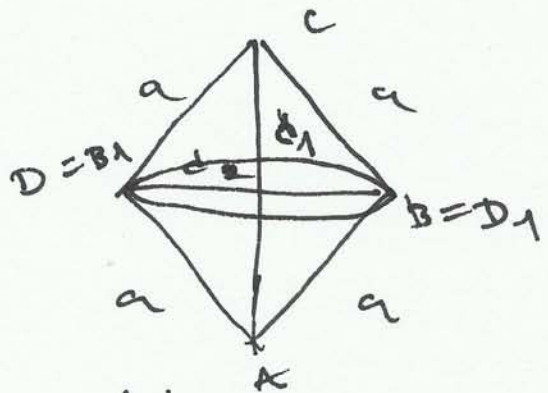
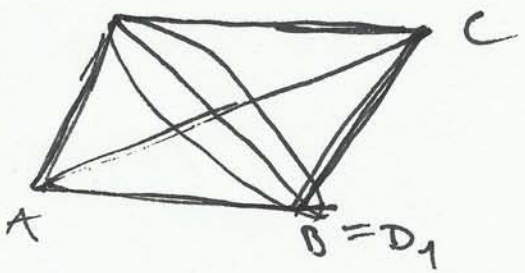
$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 =$$

$$= \left(\frac{12}{2}\right)^2 + \left(\frac{16}{2}\right)^2 =$$

$$= 6^2 + 8^2 = 100$$

$$a = 10$$

D = B1



A i C - firsre tačke

KUPA

Nastaj - dve KUPE

$$H_k = \frac{d_2}{2} = \frac{16}{2} = 8$$

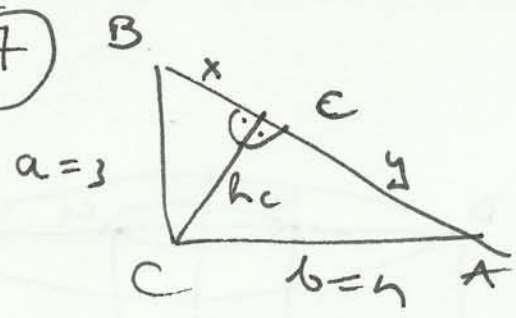
$$H_k = \frac{d_1}{2} = \frac{12}{2} = 6$$

$$S_k = a = 10$$

$$P = 2M_k = 2 \cdot \pi \cdot S_k \cdot H_k = 2 \cdot 6 \cdot \pi \cdot 10 = 120\pi$$

$$V = \frac{2 \cdot \pi \cdot S_k^2 \cdot H_k}{3} = \frac{2 \cdot 6^2 \cdot \pi \cdot 8}{3} = 192\pi$$

7



$$c^2 = a^2 + b^2 = 3^2 + 4^2 = 25$$

$$c = 5$$

$$P = \frac{ab}{2} = \frac{c h_c}{2}$$

$$P = \frac{3 \cdot 4}{2} = 6 \quad 6 = \frac{5 \cdot h_c}{2}$$

$$5 h_c = 12$$

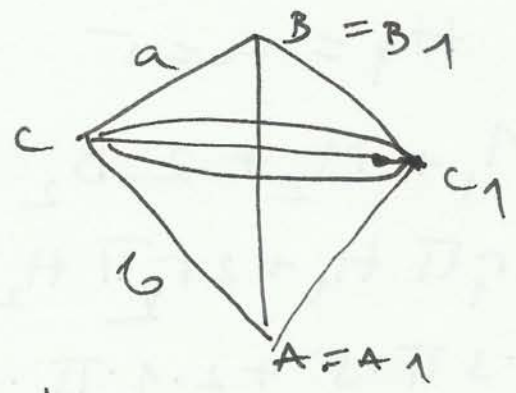
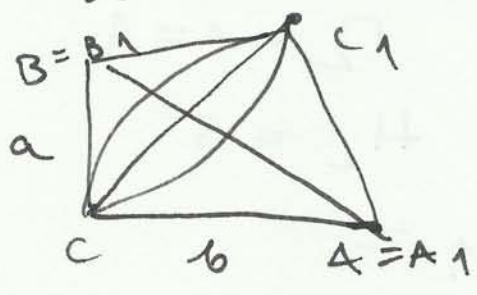
$$h_c = \frac{12}{5}$$

$$x^2 = a^2 - h_c^2$$

$$x^2 = 3^2 - \left(\frac{12}{5}\right)^2 = 9 - \frac{144}{25} = \frac{225 - 144}{25} = \frac{81}{25}$$

$$x = \frac{9}{5}$$

$$y = c - x = 5 - \frac{9}{5} = \frac{25 - 9}{5} = \frac{16}{5}$$



Dobijemo se dve KUPE

KUPA 1

KUPA 2

$$r_{k_1} = h_c = \frac{12}{5}$$

$$r_{k_2} = h_c = \frac{12}{5}$$

$$H_{k_1} = x = \frac{9}{5}$$

$$H_{k_2} = y = \frac{16}{5}$$

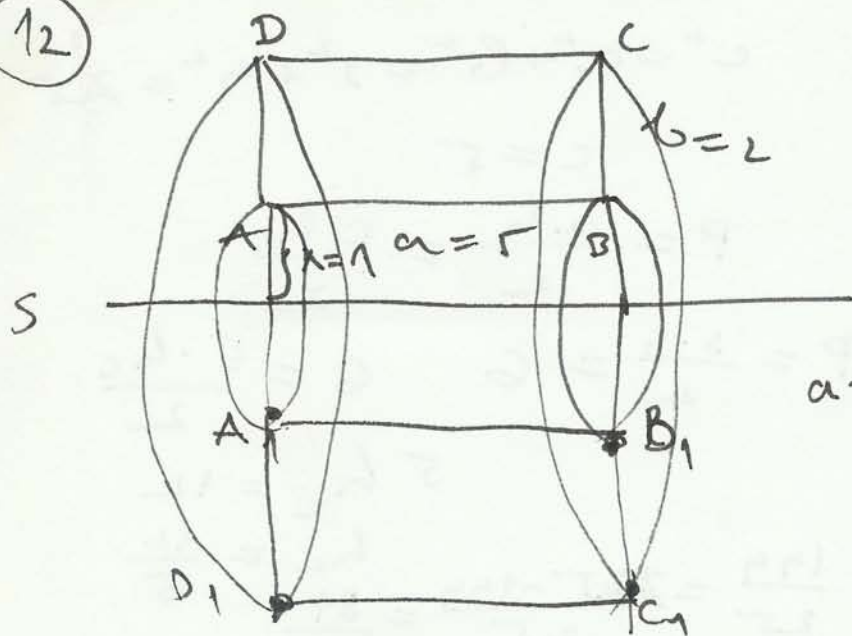
$$s = a = 3$$

$$s = b = 4$$

$$P = M_{k_1} + M_{k_2} = \frac{\sqrt{\pi} \pi s_{k_1}}{3} + \frac{\sqrt{\pi} \pi s_{k_2}}{3} = \dots$$

$$V = V_{k_1} + V_{k_2} = \frac{\frac{\sqrt{\pi} \pi H_{k_1}}{3}}{3} + \frac{\frac{\sqrt{\pi} \pi H_{k_2}}{3}}{3}$$

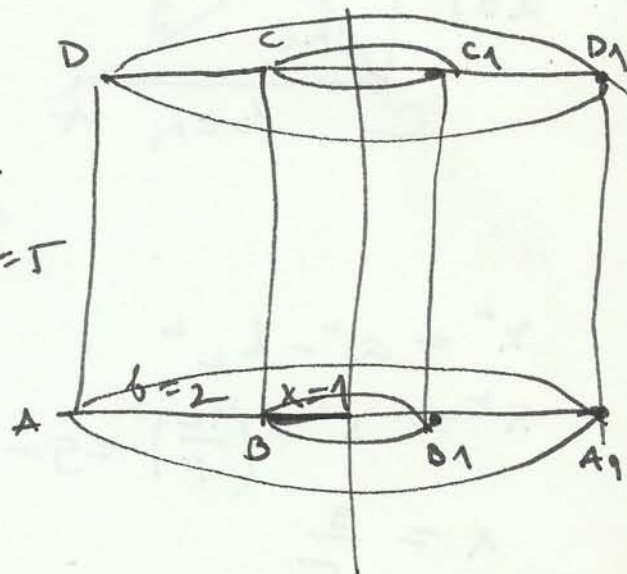
12



VELIKI VAČAK

$$\Gamma_1 = b + x = 2 + 1 = 3$$

$$H_1 = a = 5$$



MALI VAČAK

$$\Gamma_2 = x = 1$$

$$H_2 = 5$$

$$P = M_1 + M_2 + 2B_2 - 2B_1 =$$

$$= 2\Gamma_1\pi H_1 + 2\Gamma_2\pi H_2 + 2\Gamma_1^2\pi - 2\Gamma_2^2\pi =$$

$$= 2 \cdot 3\pi \cdot 5 + 2 \cdot 1 \cdot \pi \cdot 5 + 2 \cdot 3^2\pi - 2 \cdot 1^2\pi =$$

$$= 30\pi + 10\pi + 18\pi - 2\pi = 56\pi$$

$$V = V_1 - V_2 = \Gamma_1^2\pi H_1 - \Gamma_2^2\pi H_2 =$$

$$= 3^2\pi \cdot 5 - 1^2\pi \cdot 5 = 40\pi$$