

KVADRAT BINOMA

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(-a-b)^2 = (a+b)^2 = a^2 + 2ab + b^2$$

$$(-a+b)^2 = (b-a)^2 = (a-b)^2 = a^2 - 2ab + b^2$$

RAZLIKA KVADRATA

$$a^2 - b^2 = (a-b)(a+b)$$

① Ako je $P = \sqrt{5-\sqrt{24}} - \sqrt{5+\sqrt{24}}$
irracionalas P^2 .

$$\begin{aligned} P^2 &= (\sqrt{5-\sqrt{24}} - \sqrt{5+\sqrt{24}})^2 = \\ &= (\sqrt{5-\sqrt{24}})^2 - 2\sqrt{5-\sqrt{24}}\sqrt{5+\sqrt{24}} + (\sqrt{5+\sqrt{24}})^2 = \\ &= 5 - \cancel{\sqrt{24}} - 2\sqrt{5^2 - (\sqrt{24})^2} + 5 + \cancel{\sqrt{24}} = \\ &= 10 - 2\sqrt{25-24} = 10 - 2 \cdot \sqrt{1} = 10 - 2 = 8 \end{aligned}$$

② Ako je $x = 18,5$ i $y = 8,2$
irracionalas $4x^2 - 20xy + 25y^2$.

$$\begin{aligned} 4x^2 - 20xy + 25y^2 &= (2x - 5y)^2 = \\ &= (2 \cdot 18,5 - 5 \cdot 8,2)^2 = (37 - 41)^2 = \\ &= (-4)^2 = 4^2 = 16 \end{aligned}$$

③ Ako je $\sqrt{2}x - \sqrt{2}y = \sqrt{18}$
 i tražimo $\frac{\sqrt{3}}{3}x - \frac{y}{\sqrt{3}}$.

$$\begin{aligned}\sqrt{2}x - \sqrt{2}y &= \sqrt{18} \\ \sqrt{2}(x - y) &= \sqrt{18} \\ x - y &= \frac{3\sqrt{2}}{\sqrt{2}} \\ x - y &= 3\end{aligned}$$

$$\begin{aligned}\frac{\sqrt{3}}{3}x - \frac{y}{\sqrt{3}} &= \\ &= \frac{\sqrt{3}}{3}x - \frac{y}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \\ &= \frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{3}y = \\ &= \frac{\sqrt{3}}{3}(x - y) = \\ &= \frac{\sqrt{3}}{3} \cdot 3 = \sqrt{3}\end{aligned}$$

④ Ako je $a^2 + b^2 = 104$ i $ab = 20$
 naći $a - b$ i $a + b$.

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 = a^2 + b^2 - 2ab = \\ &= 104 - 2 \cdot 20 = 104 - 40 = 64 \\ a - b &= \sqrt{64} = 8\end{aligned}$$

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 = a^2 + b^2 + 2ab = \\ &= 104 + 2 \cdot 20 = 144 \\ a + b &= \sqrt{144} = 12\end{aligned}$$

⑤ Ako je $4x^2 - 25y^2 = 12$ i $2x - 5y = 3$
 naći $2x + 5y$.

$$\begin{aligned}4x^2 - 25y^2 &= (2x - 5y)(2x + 5y) \\ 12 &= 3 \cdot (2x + 5y)\end{aligned}$$

$$2x + 5y = 4$$

⑥ ZBIR BROJEVA a i b JE 35. AKO JE BROJ a ČETIRI PUTA VEĆI OD BROJA b NAĆI $a-b$.

$$\begin{array}{r} a+b=35 \\ a=4b \\ \hline 4b+b=35 \\ a=4b \\ \hline 5b=35 \\ a=4b \\ \hline b=7 \end{array}$$

$$a=4 \cdot 7 = 28$$

$$a-b=28-7$$

$$a-b=21$$

⑦ AKO JE $x \in \mathbb{N}$, $x > 1$ i AKO JE $x^2 + \frac{1}{x^2} = \frac{82}{9}$ NAĆI $x + \frac{1}{x}$ i $x - \frac{1}{x}$.

$$\begin{aligned} \left(x + \frac{1}{x}\right)^2 &= x^2 + 2 \times \frac{1}{x} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2} = \\ &= x^2 + \frac{1}{x^2} + 2 = \frac{82}{9} + 2 = \\ &= \frac{82 + 18}{9} = \frac{100}{9} \end{aligned}$$

$$x + \frac{1}{x} = \sqrt{\frac{100}{9}} = \frac{10}{3} = 3\frac{1}{3} \quad (x=3)$$

$$\begin{aligned} \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 \times \frac{1}{x} + \frac{1}{x^2} = \frac{82}{9} - 2 = \\ &= \frac{82 - 18}{9} = \frac{64}{9} \end{aligned}$$

$$x - \frac{1}{x} = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2\frac{2}{3} \quad (x=3)$$

8) Ako je $\frac{a+b}{b} = 2 - \sqrt{2}$ traži $\frac{a^2+b^2}{ab}$.

$$\frac{a+b}{b} = \frac{a}{b} + \frac{b}{b} = \frac{a}{b} + 1$$

$$\frac{a}{b} + 1 = 2 - \sqrt{2}$$

$$\frac{a}{b} = 2 - \sqrt{2} - 1$$

$$\frac{a}{b} = 1 - \sqrt{2}$$

$$\begin{aligned} \frac{b}{a} &= \frac{1}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{1 + \sqrt{2}}{1^2 - (\sqrt{2})^2} = \frac{1 + \sqrt{2}}{1 - 2} = \\ &= \frac{1 + \sqrt{2}}{-1} = -1 - \sqrt{2} \end{aligned}$$

$$\begin{aligned} \frac{a^2+b^2}{ab} &= \frac{a^2}{ab} + \frac{b^2}{ab} = \frac{a}{b} + \frac{b}{a} = \\ &= 1 - \sqrt{2} - 1 - \sqrt{2} = -2\sqrt{2} \end{aligned}$$

RAČIONALIZACIJA I MENJALACA

$$\frac{1}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{\sqrt{3} + \sqrt{2}}{3 - 2} = \frac{\sqrt{3} + \sqrt{2}}{1}$$

(dopuna do
ratlike kvadrata) = $\sqrt{3} + \sqrt{2}$

$$\begin{aligned} \frac{8}{\sqrt{7} + \sqrt{5}} \cdot \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} &= \frac{8(\sqrt{7} - \sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} = \frac{8(\sqrt{7} - \sqrt{5})}{7 - 5} = \\ &= \frac{8(\sqrt{7} - \sqrt{5})}{2} = 4(\sqrt{7} - \sqrt{5}) \end{aligned}$$

$$\frac{12}{\sqrt{10}-2} \cdot \frac{\sqrt{10}+2}{\sqrt{10}+2} = \frac{12(\sqrt{10}+2)}{(\sqrt{10})^2 - 2^2} =$$

$$= \frac{12(\sqrt{10}+2)}{10-4} = \frac{12(\sqrt{10}+2)}{6}$$

$$= 2(\sqrt{10}+2)$$

$$\frac{20}{\sqrt{11}+\sqrt{7}} \cdot \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} = \frac{20(\sqrt{11}-\sqrt{7})}{(\sqrt{11})^2 - (\sqrt{7})^2} =$$

$$= \frac{20(\sqrt{11}-\sqrt{7})}{11-7} = \frac{20(\sqrt{11}-\sqrt{7})}{4}$$

$$= 5(\sqrt{11}-\sqrt{7})$$

$$\frac{6}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{6(3+\sqrt{3})}{3^2 - (\sqrt{3})^2} = \frac{6(3+\sqrt{3})}{9-3}$$

$$= \frac{6(3+\sqrt{3})}{6} = 3+\sqrt{3}$$

RESI JEDNAČINÉ:

$$\begin{aligned} \textcircled{1} \quad x^2 - \frac{3}{7}x &= 0 \\ x \left(x - \frac{3}{7} \right) &= 0 \\ \boxed{x=0} \quad x - \frac{3}{7} &= 0 \\ \boxed{x = \frac{3}{7}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 5x^2 - 30x + 45 &= 0 \\ 5(x^2 - 6x + 9) &= 0 \\ 5(x-3)^2 &= 0 \\ x-3 &= 0 \\ \boxed{x=3} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad (x+1)^2 &= 4 \\ \swarrow \quad \searrow & \\ x+1=2 \quad x+1=-2 & \\ \boxed{x=1} \quad \boxed{x=-3} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad (x-5)^2 - (x-4)^2 &= 3(5-6x) \\ x^2 - 10x + 25 - (x^2 - 8x + 16) &= 15 - 18x \\ \cancel{x^2} - 10x + 25 - \cancel{x^2} + 8x - 16 &= 15 - 18x \\ -2x + 9 &= 15 - 18x \\ 18x - 2x &= 15 - 9 \\ 16x &= 6 \end{aligned}$$

$$x = \frac{6}{16} = \frac{3}{8}$$

$$\begin{aligned} \textcircled{5} \quad (3a-5)(5+3a) - 9(a-2)(a+1) &= -5 \\ 9a^2 - 25 - 9(a^2 + a - 2a - 2) &= -5 \\ \cancel{9a^2} - 25 - \cancel{9a^2} - \cancel{9a} + 18a + 18 &= -5 \\ 9a - 7 &= -5 \\ 9a &= -5 + 7 \quad a = \frac{2}{9} \\ 9a &= 2 \end{aligned}$$

$$\textcircled{6} \quad \sqrt{x^2} = x+5$$

$$|x| = x+5$$

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$\text{I } x \geq 0$$

$$|x| = x+5$$

$$x = x+5$$

$$x-x = 5$$

$$\cancel{0} = 5 \quad |$$

$$, x < 0$$

$$\text{II } x < 0$$

$$|x| = x+5$$

$$-x = x+5$$

$$-x-x = 5$$

$$-2x = 5$$

$$\boxed{x = -\frac{5}{2} = -2\frac{1}{2}}$$

$\textcircled{7}$

$$\frac{2008^{2007} + 2008^{2008}}{2009} = 2008^x$$

$$\frac{2008^{2007}(1+2008)}{2009} = 2008^x$$

$$\frac{2008^{2007} \cdot \cancel{2009}}{\cancel{2009}} = 2008^x$$

$$x = 2007$$

┌
└ INVLACIJE

manji stepen

$$x^{\textcircled{5}} + x^{\textcircled{3}} = x^{\textcircled{3}}(x+1)$$

$$x^3 - x^5 = x^3(1-x^2)$$

$$x^{16} + x^{17} = x^{16}(1+x)$$

$$\begin{aligned}
 8 & 8^8 + (4^4)^x = 2^{25} \\
 & (2^3)^8 + ((2^2)^4)^x = 2^{25} \\
 & 2^{24} + (2^8)^x = 2^{25} \\
 & 2^{24} + 2^{8x} = 2^{25} \\
 & 2^{8x} = 2^{25} - 2^{24} \\
 & 2^{8x} = 2^{24}(2-1) \\
 & 2^{8x} = 2^{24} \\
 & 8x = 24 \\
 & \boxed{x = 3}
 \end{aligned}$$

9) ODREDI a TAKO DA JEDNAČINE BUDU
EKVIVALENTNE (IMAJU ISTA REŠENJA!)

$$a(x-1)+3=-1$$

$$a(-1-1)+3=-1$$

$$-2a+3=-1$$

$$-2a=-1-3$$

$$-2a=-4$$

$$\boxed{a=2}$$

$$\frac{1-5x}{5} - 2 = \frac{3}{8}(1+x)$$

$$8(1-5x) - 2 \cdot 40 = 5 \cdot 3(1+x) \quad / \cdot 40$$

$$8 - 72x - 80 = 15 + 15x$$

$$-15x - 72x = 15 + 80 - 8$$

$$-87x = 87$$

$$\boxed{x = -1} \quad (\text{PRVO NAĐEM}) \\ \text{REŠENJE}$$